

Constraining Warm Inflation with the Cosmic Microwave Background

Lisa M. H. Hall* and Ian G. Moss†

School of Mathematics and Statistics, University of Newcastle Upon Tyne, NE1 7RU, UK

Arjun Berera‡

School of Physics, University of Edinburgh, Edinburgh, EH9 3JZ, U.K.

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We discuss the spectrum of scalar density perturbations from warm inflation when the friction coefficient Γ in the inflaton equation is dependent on the inflaton field. The spectral index of scalar fluctuations depends on a new slow-roll parameter constructed from Γ . A numerical integration of the perturbation equations is performed for a model of warm inflation and gives a good fit to the WMAP data for reasonable values of the model's parameters.

The inflationary paradigm has proved to be the most successful idea for providing the seeds of large scale structure in the universe. A plethora of variants of the inflationary model exist today and in most of these models, radiation is red-shifted during the expansion, resulting in a vacuum-dominated Universe, giving the name supercooled inflation. A subsequent reheating period is invoked to end inflation and fill the universe with radiation [1, 2, 3, 4]. In an alternative picture, termed warm inflation, dissipative effects, arising from inflaton interactions, are important during the inflation period, so that radiation production occurs concurrently with inflationary expansion [5, 6, 7, 8, 9, 10].

In this letter we shall demonstrate how it is possible to constrain a particular type of warm inflationary model using Cosmic Microwave Background observations. The model which we consider is based on broken supersymmetry and has the property that the inflaton has a two stage decay chain $\phi \rightarrow \chi \rightarrow \psi$ [11, 12]. This two stage decay occurs for a wide range of inflationary models and provides a robust realisation of warm inflation. In these models the friction coefficient Γ in the inflaton equation is a function of the inflaton field and this feature leads to some new effects on the perturbation spectrum, which we describe here.

The development of cosmological perturbations in warm inflation has been investigated previously by several authors. Taylor and Berera [13] have analysed the perturbation spectra by identifying the small scale thermal fluctuations with cosmological perturbations when the fluctuations cross the horizon. This technique is good for order of magnitude estimates. For a more accurate treatment of the density perturbations it is necessary to use the cosmological perturbation equations, which can be found in [14, 15, 16]. A numerical code has recently been developed which can solve the perturbation equations for any form of friction coefficient [17].

We shall consider the case where the radiation produced by the inflaton field consists of low mass particles ψ which are approximately thermalised. In this case the density fluctuations arise from thermal, rather than vacuum, fluctuations [5, 7, 8, 9, 17, 18]. The intermediate particle χ is not in a thermal state and thermal corrections to the inflaton potential are suppressed. This allows us to use the zero-temperature potential during inflation. The effects of thermal corrections will be discussed in future work.

Assuming the early universe to be in homogeneous expansion with expansion rate $H(t)$, the evolution of the inflaton field ϕ is described by the equation

$$\ddot{\phi} + (3H + \Gamma)\dot{\phi} + V_{,\phi} = 0 \quad (1)$$

where $\Gamma(\phi)$ is the damping term and $V(\phi)$ is the inflaton potential[5, 8, 19]. The relative strength of the thermal damping compared to the expansion damping can be expressed by a parameter r ,

$$r = \frac{\Gamma}{3H} \quad (2)$$

The strong dissipation form of warm inflation occurs for large values of r , as opposed to supercooled inflation in which r can be neglected.

The dissipation of the inflaton's motion is associated with the production of entropy. Energy-momentum conservation implies that

$$T(\dot{s} + 3Hs) = \Gamma\dot{\phi}^2. \quad (3)$$

*Electronic address: lisa.hall@ncl.ac.uk

†Electronic address: ian.moss@ncl.ac.uk

‡Electronic address: ab@ph.ed.ac.uk

where s is the entropy density. For the situation under discussion, the entropy density is related to the energy density in the radiation ρ_r by $\rho_r = \frac{3}{4}Ts$, where T is the temperature, $s = \frac{\pi^2}{30}g_*T^3$ and g_* is the effective number of degrees of freedom in the radiation. In more general situations, using the entropy density is preferable to using the radiation energy density [17].

The zero curvature Friedman equation completes the set of differential equations for ϕ , T and the scale factor a ,

$$3H^2 = 8\pi G(\frac{1}{2}\dot{\phi}^2 + V + \frac{3}{4}Ts). \quad (4)$$

The slow-roll approximation consists of neglecting terms in the preceding equations with the highest order in time derivatives,

$$\dot{\phi} = \frac{-V_{,\phi}}{3H(1+r)} \quad (5)$$

$$Ts = r\dot{\phi}^2 \quad (6)$$

$$3H^2 = 8\pi GV. \quad (7)$$

Slow-roll automatically implies inflation, $\ddot{a} > 0$.

The consistency of the slow-roll approximation is governed by a set of slow-roll parameters. Warm inflation has extra slow-roll parameters in addition to the usual set for supercooled inflation due to the presence of the damping term Γ . The ‘leading order’ slow-roll parameters are,

$$\epsilon = \frac{1}{16\pi G} \left(\frac{V_{,\phi}}{V} \right)^2, \quad \eta = \frac{1}{8\pi G} \left(\frac{V_{,\phi\phi}}{V} \right), \quad \beta = \frac{1}{8\pi G} \left(\frac{\Gamma_{,\phi}V_{,\phi}}{\Gamma V} \right) \quad (8)$$

The first two parameters are the standard ones introduced for supercooled inflation [20, 21, 22, 23]. A new parameter is required when there is ϕ dependence in the damping term. A further slow-roll parameter is required when the potential has thermal corrections [17]. The slow-roll approximation is valid when all of these parameters are smaller than $1+r$. In supercooled inflation, the condition is tighter and the slow roll parameters have to be smaller than 1.

In the warm inflationary regime where $r \gg 1$, the rate of change of various slowly varying parameters are given by differentiating equations (5-7),

$$\frac{1}{H} \frac{d \ln H}{dt} = -\frac{1}{r}\epsilon \quad (9)$$

$$\frac{1}{H} \frac{d \ln \dot{\phi}}{dt} = -\frac{1}{r}(\eta - \beta) \quad (10)$$

$$\frac{1}{H} \frac{d \ln Ts}{dt} = -\frac{1}{r}(2\eta - \beta - \epsilon) \quad (11)$$

Inflation ends at the time when $\epsilon = r$, since then $\ddot{a} = \dot{H} + H^2 = 0$.

Cosmological perturbations are best described in terms of gauge invariant quantities, the curvature perturbation \mathcal{R} [24], and the entropy perturbation e [25]. An analytic approximation to the density perturbation amplitude for wave number k can be obtained by matching the classical perturbations to the thermal fluctuation amplitude at the crossing time $k = aH$,

$$\mathcal{P}_{\mathcal{R}} \sim \left(\frac{\pi}{4} \right)^{1/2} \frac{H^{5/2}\Gamma^{1/2}T}{\dot{\phi}^2}. \quad (12)$$

This result is analogous to the result $\mathcal{P}_{\mathcal{R}} = H^4/\dot{\phi}^2$ for supercooled inflation.

The spectral index n_s is defined by

$$n_s - 1 = \frac{\partial \ln \mathcal{P}_{\mathcal{R}}}{\partial \ln k}. \quad (13)$$

The slow-roll equations (9-11) enable us to express the spectral index for the amplitude (12) in terms of slow-roll parameters,

$$n_s - 1 = \frac{1}{r} \left(-\frac{9}{4}\epsilon + \frac{3}{2}\eta - \frac{9}{4}\beta \right). \quad (14)$$

The first two terms agree with reference [18]. The β term shows the dependence of the spectrum on the gradient of the damping term. For comparison, the spectral index for standard, or supercooled inflation, is $n_s = 1 - 6\epsilon + 2\eta$ [26].

There are two second-order slow-roll parameters

$$\zeta^2 = \frac{1}{(8\pi G)^2} \left(\frac{V_{,\phi} V_{,\phi\phi\phi}}{V^2} \right), \quad \gamma = \frac{1}{8\pi G} \left(\frac{\Gamma_{,\phi\phi}}{\Gamma} \right). \quad (15)$$

With these we can obtain an expression for the slope of the spectral index,

$$\frac{dn_s}{d \ln k} = \frac{1}{r^2} \left(-\frac{9}{2}\beta^2 - \frac{27}{4}\epsilon^2 - \frac{9}{2}\epsilon\beta + \frac{15}{4}\eta\beta + 6\epsilon\eta - \frac{3}{2}\zeta^2 + \frac{9}{2}\gamma\epsilon \right). \quad (16)$$

The first three terms form a negative definite combination, but other terms, including $-\zeta^2$, can be positive.

The effect of β in combination with ‘new inflation’ potentials is particularly interesting. Such potentials have a maximum at $\phi = 0$ and a minimum at $\phi = \phi_0$ [2, 3, 4]. If we take $\Gamma \propto \phi^b$, then the behaviour of n_s in the vicinity of the two extrema can be determined

$$n_s - 1 \rightarrow \begin{cases} -|\eta|(2-3b) & \phi \rightarrow 0 \\ -\infty & \phi \rightarrow \phi_0 \end{cases}. \quad (17)$$

Therefore, if $b > 2/3$, there will be a value ϕ_m at which $n_s = 1$ and n_s is decreasing. The density perturbation amplitude has a maximum at the wave number which crossed the horizon when $\phi = \phi_m$, and the spectrum runs from blue to red as the wave number increases. The possibility of a spectrum which runs from blue to red is particularly interesting because it is not commonly seen in inflationary models, which typically predict red spectra. (For examples of inflation with blue spectra, see [27, 28]).

Now consider the spectrum of density perturbations for a model with interaction Lagrangians $\frac{1}{2}g^2\phi^2\chi^2$ and $h\chi\bar{\psi}\psi$. The friction term can be approximated by [11, 12]

$$\Gamma \approx \frac{g^3 h^2 \phi}{256\pi^2}. \quad (18)$$

Warm inflation occurs when $g \approx h \approx 0.1$. For coupling constants of this magnitude, quantum loop corrections to the inflaton must be suppressed by an underlying supersymmetry to prevent them violating the slow-roll conditions. The supersymmetry is broken during inflation, either spontaneously, or by soft supersymmetry breaking masses. (These soft terms are not necessarily the same soft terms which occur at the TeV scale).

We take the soft supersymmetry breaking option, with inflaton potential,

$$V(\phi) = \frac{1}{2}\mu_0^2 \left(\phi^2 \log \left(\frac{\phi^2}{\phi_0^2} \right) + \phi_0^2 - \phi^2 \right). \quad (19)$$

The damping term Γ , will be parameterised by

$$\Gamma = \Gamma_0 \frac{\phi}{\phi_0}. \quad (20)$$

With this combination, prolonged inflation (sixty e-folds or more) can occur for values of ϕ_0 smaller than the Planck scale m_p . The running of the spectral index (16) is always negative. By contrast, in the supercooled limit $\Gamma_0 = 0$, a prolonged period of inflation requires ϕ_0 to be at least $2m_p$ and it may not be consistent to ignore the effects of quantum gravity. The density fluctuation amplitude for supercooled inflation requires μ_0 to be around 10^{13} GeV.

Our numerical code solves the homogeneous background and perturbation equations simultaneously, with no assumption of slow-roll. The reader is referred to [17] for further details. The code solves for the primordial power spectrum with the input parameters μ_0 , ϕ_0 and Γ_0 defined above. In order to compare to WMAP data, power law behaviour of the spectrum is assumed about a scale $k_0 = 0.002 Mpc^{-1}$. The power spectrum is then characterised by the index, n_s and spectral running $dn/d \ln k$.

We compute the microwave anisotropies using the CAMB code [29] and run the WMAP likelihood code [30] in order to compute the likelihood fit of the model to WMAP data. Since at present we are interested in isolating the behaviour of the above warm inflationary parameters, μ_0 , ϕ_0 and Γ_0 , we assume priors for the cosmological parameters (Ω_Λ , Ω_b , Ω_m etc.). The assumed parameter values are given in table I and are taken from Bennett et al.[31]. The tensor components are assumed to be negligible.

For given values of μ_0 and Γ_0 , the value of ϕ_0 is chosen to normalise the amplitude of the power spectrum to the WMAP value A_{WMAP} at $k = k_0$. (In practice, we save some computation time by fixing the value of ϕ_0 and using

TABLE I: Cosmological parameters taken from Bennett[31]. These priors are used as inputs into the CAMB code.

Parameter	Symbol	Value
Dark energy density	Ω_Λ	0.73
Baryon density	Ω_b	0.0044
Matter density	Ω_m	0.27
Light neutrino density	Ω_ν	0
Spatial curvature density	Ω_k	0
Hubble constant	h	0.71

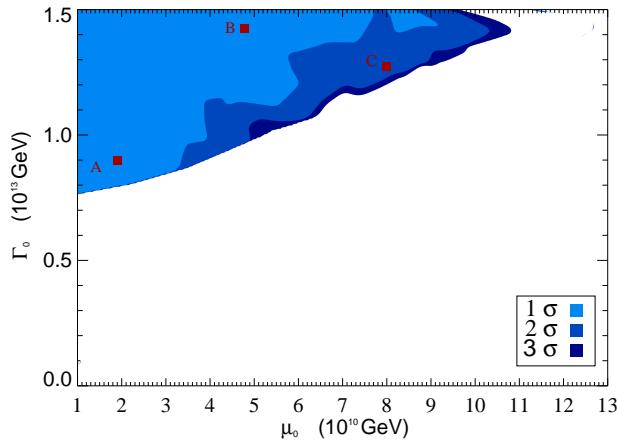


FIG. 1: The 1σ , 2σ and 3σ bounds on the parameters for the potential and friction term, μ_0 and Γ_0 as calculated with CAMB in conjunction with the WMAP likelihood code. Values for the spectral index and spectral running at the points marked A, B and C are given in table II.

renormalisation to rescale the model to the correct amplitude.) We have restricted the range of ϕ_0 to be less than the Planck scale.

Two inflationary parameters remain to be investigated, namely μ_0 and Γ_0 . The 1σ , 2σ and 3σ bounds for this parameter space are displayed in figure 1. Note that there is no supercooled inflationary limit ($\Gamma_0 \rightarrow 0$) in figure 1 because the density fluctuation amplitude for supercooled inflation is too small in the parameter range shown.

The 3σ limit extends well beyond the upper edge of the figure $\Gamma_0 > 1.5 \times 10^{13} \text{ GeV}$. Consideration of the friction term (18) shows that a value of $\Gamma_0 = 1.5 \times 10^{13} \text{ GeV}$ corresponds to couplings, $g \approx h \approx 0.2$. As g and h increase further still (Γ_0 increasing), we go beyond the perturbative regime in which the potential which we are using is valid. We also expect thermal corrections to the potential to be important when the coupling constants reach this magnitude, and we shall return to this issue in future work.

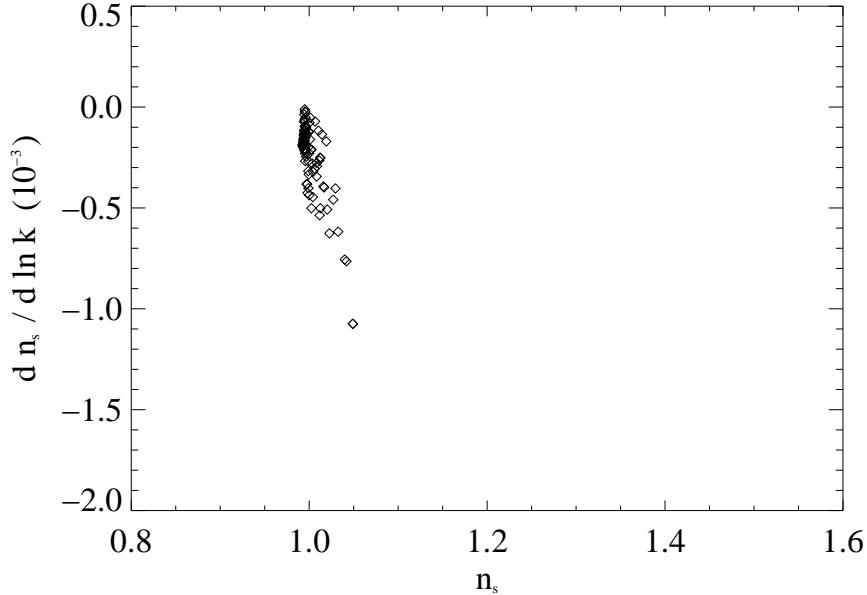
At points A, B and C in figure 1, the spectral index comes out very close to $n_s = 1$ with a very small amount of spectral running (see table II). The running of the spectrum is negative, as we predicted above, but rather small compared to some values which have been discussed in connection with the WMAP data.

Typical properties of the spectrum can be understood by analysing the spectra produced by a uniformly distributed set of parameter values as in figure 2. The parameter values are only included if inflation lasts long enough to affect the large scale structure. The spectra produced by these uniform parameter points are all within a small range around $n_s = 1$ at a scale $k_0 = 0.002 \text{ Mpc}^{-1}$, with small spectral running. In contrast, the WMAP data can accommodate large spectral running if there are relatively large departures from $n_s = 1$ [32], and the observations are not yet accurate enough to provide very strong restrictions on the parameter values.

We conclude by summing up the main results. For warm inflation, a third slow-roll parameter is introduced due to the dependence of the spectrum on the dissipation term, Γ . For inflation with new inflation potentials, it has been shown that, in warm inflation, one expects the spectral index to run from blue to red as the wavenumber increases. This behaviour is particularly interesting in the light of the first year WMAP data, in which such a running is hinted at. For a specific potential, representative of supersymmetry breaking, warm inflation has been compared to the WMAP data and a good fit was found for a large range of parameter values. The model produces spectra with n_s close to one and a very small negative running of the index. Future observations of the Cosmic Microwave Background

TABLE II: Spectral index and running for points A, B and C in figure 1

	n_s	$d n_s / d \ln k$
A	0.995	-2.1×10^{-4}
B	0.994	-3.7×10^{-5}
C	1.004	-3.2×10^{-4}

FIG. 2: These points represent the spectra produced for a uniformly distributed selection of parameter values μ_0 and Γ_0 . Note that the vertical axis is in units of 10^{-3} .

are needed to detect such small slopes in the spectrum and produce stronger constraints on the parameters.

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- [1] A. H. Guth, Phys. Rev. D **23**, 347 (1981).
- [2] A. Linde, Phys. Lett. **108B**, 389 (1982).
- [3] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).
- [4] S. W. Hawking and I. G. Moss, Phys. Lett. **110B**, 35 (1982).
- [5] I. G. Moss, Phys. lett. **154B**, 120 (1985).
- [6] J. Yokoyama and K. Maeda, Phys. lett. B **207**, 31 (1988).
- [7] A. Berera and L. Z. Fang, Phys. Rev. lett. **74**, 1912 (1995).
- [8] A. Berera, Phys. Rev. lett. **75**, 3218 (1995).
- [9] A. Berera, Phys. Rev. D **54**, 2519 (1996).
- [10] A. Berera, Phys. Rev. D **55**, 3346 (1997).
- [11] A. Berera and R. O. Ramos, Phys. Lett. B **567**, 294 (2003).
- [12] A. Berera and R. O. Ramos, [hep-ph/0308211](#) (2003).
- [13] A. N. Taylor and A. Berera, Phys. Rev. D **62**, 083517 (2000).
- [14] W. Lee and L.-Z. Fang, Phys. Rev. D **59**, 083503 (1999).
- [15] H. P. de Oliveira and S. E. Joras, Phys. Rev. D **64**, 063613 (2001).

- [16] J. chan Hwang and H. Noh, *Class. Quantum Grav.* **19**, 527 (2002).
- [17] L. M. H. Hall, I. G. Moss, and A. Berera, [astro-ph/0305015](#) (2003).
- [18] A. Berera, *Nucl. Phys. B* **585**, 666 (2000).
- [19] A. Hosoya and M. aki Sakagami, *Phys. Rev. D* **29**, 2228 (1984).
- [20] A. R. Liddle, P. Parsons, and J. D. Barrow, *Phys. Rev. D* **50**, 7222 (1994).
- [21] E. D. Stewart, *Phys. Rev. D* **65**, 103508 (2002).
- [22] S. Habib, K. Heitmann, G. Jungman, and C. Molina-Paris, *Phys. Rev. Lett.* **89**, 281301 (2002).
- [23] J. Martin and D. J. Schwarz, *Phys. Rev. D* **67**, 083512 (2003).
- [24] V. N. Lukash, *Sov. Phys. JETP* **52**, 807 (1980).
- [25] H. Kodama and M. Sasaki, *Prog. Theor. Phys. Suppl.* **78**, 1 (1984).
- [26] A. R. Liddle and D. H. Lyth, *Phys. Lett. B* **291**, 391 (1992).
- [27] E. J. Copeland, A. R. Liddle, A. R. Lyth, D. H. Stewart, and D. Wands, *Phys. Rev. D* **49**, 6410 (1994).
- [28] A. D. Linde and A. Riotto, *Phys. Rev. D* **56**, 1841 (1997).
- [29] A. Lewis, A. Challinor, and A. Lasenby, *Astrophys. J.* **538**, 473 (2000).
- [30] L. Verde et al., *Astrophys. J. Suppl.* **148**, 195 (2003).
- [31] C. L. Bennett et al., *Astrophys. J. Suppl.* **148**, 1 (2003).
- [32] H.V.Peiris et al., *Astrophys. J. Suppl.* **148**, 213 (2003).